STAT 821

Homework #3
Due on Janurary 31, 2012 at noon

NOTE: The following problems are from Chapter 1 of the textbook.

1) Suppose $X_1, \cdots, X_n$ is a sample from a population with one of the following densities. Find a sufficient statistic for θ.
   (a) Beta density: $p_\theta(x) = \theta \cdot x^{\theta-1}$, where $0 < x < 1$ and $\theta > 0$
   (b) Weibull density: $p_\theta(x) = \theta \cdot ax^{a-1}\exp(-\theta x^a)$, where a is a positive constant, $x > 0$ and $\theta > 0$
   (c) Pareto density: $p_\theta(x) = \theta \cdot a^\theta / x^{\theta+1}$, where a is a positive constant, $x > a$ and $\theta > 0$
   $0 < x < 1$ and $\theta > 0$

2) **(Kolmogorov's Theorem)** Suppose the natural parameter space Θ is finite.
   (a) Suppose that a statistic $T(X)$ has the property that for any prior distribution on θ, the posterior distribution on θ depends on X only through $T(X)$. Show that $T(X)$ is sufficient.
   (b) Conversely show that if $T(X)$ is sufficient, then for any prior distribution, the posterior distribution depends on X only through $T(X)$.

3) **Problem 6.7**

4) **Problem 6.20**

5) **Problem 6.21**